

XVI Portuguese Category Seminar

Programme & abstracts

Sala Pedro Nunes
Department of Mathematics
University of Coimbra

Programme

Sala Pedro Nunes · All times local to Coimbra (WET / UTC+0).

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Venue and registration

All talks take place in Sala Pedro Nunes at the Department of Mathematics, University of Coimbra, Largo D. Dinis, 3000-143 Coimbra. The main entrance opens directly onto the first floor.

On-site registration is on Thursday, 12 November, 15:05–15:30, in front of Sala Pedro Nunes. Participation in the seminar is free.

If you selected “Yes” or “Yes + 1” for Saturday lunch, please pay **in cash during registration**. Thursday and Friday lunches are by individual arrangement.

Coffee breaks take place on the second floor. Saturday’s group photo is at 11:00, immediately before the coffee break. The conference lunch follows the final talk at 12:30.

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Thursday 12 November | 15:30–16:00

A topos for étale-finite Heyting algebras

Marco Abbadini

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Which lattices can arise as lattices of sentences, modulo provable equivalence, of intuitionistic higher-order theories? Categorically, this asks which Heyting algebras are *topos-admissible*: isomorphic to the lattice of subobjects of the terminal object of an elementary topos, also called its lattice of truth values. The question whether every Heyting algebra is topos-admissible appeared in print in a 1992 paper by Pitts [1], but dates back at least to 1980, when Pitts learned it from Freyd.

Recently, Xu and Ye answered this question negatively, showing that the free Heyting algebra on two generators is not topos-admissible [2]. This leaves open the problem of finding an intrinsic characterisation of the Heyting algebras that can occur as lattices of truth values.

Known positive cases include complete Heyting algebras (frames), via sheaf toposes, and Boolean algebras, via a construction due to Freyd. In joint work with Rodrigo Almeida and Igor Arrieta [3], we extend Freyd's construction to all *étale-finite Heyting algebras*, a class introduced by Kuznetsov. These algebras satisfy a generalisation of the law of excluded middle relative to a finite Heyting subalgebra, and include examples that are neither Boolean nor complete. Using Esakia duality, we construct, for each such algebra, an elementary topos of certain compact étale spaces whose lattice of truth values is isomorphic to the given algebra.

Keywords: Elementary toposes; Heyting algebras; Esakia duality; étale-finite algebras
Subject area: Categorical logic and topos theory
MSC2020: 03G30 (primary); 18B25, 06D20, 06D50 (secondary)

References

[1] A. M. Pitts, *On an interpretation of second order quantification in first order intuitionistic propositional logic*, Journal of Symbolic Logic **57**(1), 33–52, 1992.

[2] L. Ye and Y. Xu, *Failure of higher-order truth within intuitionistic propositional logic*, preprint, 2026. arXiv:2608.26874.

[3] M. Abbadini, R. N. Almeida and I. Arrieta, *A topos for étale-finite Heyting algebras*, preprint, 2026. arXiv:2606.03861.

Thursday 12 November | 16:30–16:50

"2-nil or not 2-nil, that is the question": a characterization of two-step nilpotent varieties

Logan Geenis

IRMP, UCLouvain, Belgium

What if William Shakespeare had been a mathematician? His most famous quote might have been: "2-nil or not 2-nil?" The aim of my talk is to answer this mathematical variant of Hamlet's question. To do so, I will present a theorem that gives a precise characterization of two-step nilpotent varieties in the sense of universal algebra.

In the abelian case, which amounts to one-step nilpotency, the corresponding answer is provided by Freyd's theorem: abelian varieties are, up to equivalence, precisely categories of modules over rings [3]. Our theorem gives a quadratic counterpart: a finitary, single-sorted variety is two-step nilpotent if and only if it is equivalent to the category of right modules over a square ring [4].

Here, a semi-abelian variety is two-step nilpotent when its ternary cosmash products vanish, or equivalently when its identity functor is quadratic. Square rings, introduced by Baues, Hartl and Pirashvili in metastable homotopy theory [1], thus extend module theory to a non-abelian setting. Similarly to abelian and semi-abelian categories, the prototypical example is the category of groups of nilpotency class at most two.

Our construction is explicit: the square ring is built from the free algebras on one and two generators, while normal forms for free objects provide concrete tools for calculations. The underlying abelian theory also remains accessible: the ordinary ring governing the abelian core is encoded in the square ring and recovered through a natural quotient construction.

The recognition theorem of Deval, Hartl and Van der Linden [2] gives a further application. Together, the two results realize every symmetric, bicocontinuous bifunctor on an abelian variety as a cosmash product of abelian objects in a category of modules over a suitable square ring. This provides a concrete algebraic framework for investigating the non-abelian structures behind tensor-like products.

Applications of our theorem and the study of the categorical properties of square rings will be presented in future work.

This is joint work in progress with Manfred Hartl and Tim Van der Linden.

Keywords: Semi-abelian category, variety of algebras, nilpotent object, cosmash product, cross-effect, polynomial functor, square ring, nearring.
Subject area: Categorical Algebra.
MSC2020: 18E13, 18C10, 08B05, 16Y30, 18E05, 20F18.

References

[1] H.-J. Baues, M. Hartl, and T. Pirashvili, Quadratic categories and square rings, *J. Pure Appl. Algebra* **122** (1997), no. 1-2, 1–40.

[2] B. S. Deval, M. Hartl, and T. Van der Linden, Intrinsic tensor products and a Ganea-type extension of the five-term exact sequence, *J. Algebra* **710** (2027), 645–715.

[3] P. Freyd, *Abelian categories*, Harper and Row, New York, 1964, Republished in: Reprints in Theory and Applications of Categories, no. 3 (2003).

[4] L. Geenis, M. Hartl and T. Van der Linden, Two-step nilpotent varieties as categories of modules over a square ring, preprint, (2026).

Friday 13 November | 09:30–10:15

A new characterization of arithmetical categories and Pixley's theorem

Marino Gran

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We characterize the arithmetical categories [1] - the exact Mal'tsev categories whose lattices of congruences are distributive - by a simple condition on some suitable pushouts and a finite limit: a regular category is exact arithmetical if and only if, for every triple of regular epimorphisms with common domain, the pairwise pushouts exist and the comparison morphism to the limit of the diagram they form is a regular epimorphism. For a regular category, the validity of this condition for pairs and for triples is equivalent to its validity for n-tuples for all $n \geq 2$, and characterizes arithmetical categories among regular ones. For pairs alone, that is when $n = 2$, the condition reduces to the characterization of exact Mal'tsev categories among regular ones, due to Carboni, Kelly and Pedicchio [2]. For n-tuples with $n \geq 4$ nothing further is obtained: arithmeticity is the third and last rung of that ladder. The proof rests on a categorical form of the Chinese Remainder Theorem; in the exact Mal'tsev context this form of the theorem lies close to results of Hoefnagel on majority categories [3]. As a direct application, we give a new proof of Pixley's characterization of arithmetical varieties [4] using a suitable diagram of free algebras. The main results presented in this talk are proved in the preprint [5].

Keywords: Arithmetical category; Pixley's theorem; the Chinese Remainder Theorem
Subject area: Category theory and universal algebra
MSC2020: 18E13, 08B10, 18E08

References

[1] M.C. Pedicchio, *Arithmetical categories and commutator theory*, Applied Categorical Structures **4**, 297–305, 1996.

[2] A. Carboni, G. M. Kelly and M. C. Pedicchio, *Some remarks on Maltsev and Goursat categories*, Applied Categorical Structures **1**, 385–421, 1993.

[3] M. Hoefnagel, *Characterizations of majority categories*, Applied Categorical Structures **28**, 113–134, 2020.

[4] A. F. Pixley, *Distributivity and permutability of congruence relations in equational classes of algebras*, Proceedings of the American Mathematical Society **14** 105–109, 1963.

[5] M. Gran, *A new characterization of arithmetical categories and Pixley's theorem*, arXiv:2609.06248, preprint, 2026.

Friday 13 November | 10:15–11:00

How protomodular is your favourite category?

Maria Manuel Clementino[1], Diana Rodelo[2]

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[2]Department of Mathematics, University of the Algarve, Portugal

Presenting author: Diana Rodelo

The aim of this talk is to investigate a relative notion of protomodularity. We say that a finitely complete category C satisfies the $\mathcal{M}$-*version of protomodularity* when the change-of-base functors (of the fibration of points) are conservative with respect to a class $\mathcal{M}$ of morphisms of C. When $\mathcal{M}$ is the class of all (mono)morphisms of C, this notion is precisely that of an "ordinary" protomodular category. We analyse which conditions on $\mathcal{M}$ are needed to obtain the relative $\mathcal{M}$-versions of some well-known properties of protomodular categories, such as the Split Short Five Lemma (when C is also pointed) and the relation with Mal'tsev categories. We then investigate the $\mathcal{M}$-version of protomodularity for different subclasses of (mono)morphisms in C such as: isomorphisms, strong monomorphisms, split monomorphisms, protosplit monomorphisms (i.e. kernels of split epimorphisms) and fully faithful monomorphisms in an Ord-enriched context. This leads to diverse examples of categories satisfying the $\mathcal{M}$-version of protomodularity for different choices of $\mathcal{M}$. None of these examples are protomodular categories in the usual sense, yet each retains a protomodular "flavour", depending on the choice of $\mathcal{M}$.

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Keywords: protomodular categories; Mal'tsev categories; categories enriched in the category of preorders
Subject area: Categorical Algebra

References

[1] M. M. Clementino, D. Rodelo, *How protomodular is your favourite category?*, arXiv:2610.09739, DMUC preprint.

Friday 13 November | 11:30–11:50

On the exactness of categories of internal groups

David Forsman

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The category $\mathrm{Hopf}_k^{\mathrm{coc}}$ of cocommutative Hopf algebras over a field k forms a semi-abelian, in particular Barr-exact, category [2]. It is the category $\mathsf{Grp}(\mathcal{C})$ of internal groups in $\mathcal{C} = \mathsf{Comon}_k^{\mathrm{coc}}$, which is itself *not* exact. The argument for exactness requires a classical result by Newman [3].

Let $\mathcal{C}$ be a finitely complete category with reflexive coequalizers that are preserved by the binary product componentwise. For an internal group h, let $\mathcal{C}^h$ be the category of h-actions. There is a surjective Galois connection $\mathrm{Sub}_{\mathsf{Grp}}(h) \rightleftarrows \mathrm{Quot}_{\mathcal{C}^h}(h)$ from subgroups $g \leq h$ to regular quotients of h in $\mathcal{C}^h$. A subgroup g is sent to its coset quotient $h \to h/g$, the coequalizer of the *coset (equivalence) relation* $h \times g \rightrightarrows h$, and a regular quotient $q\colon h \to m$ to the stabilizer $q^{-1}(q(e))$ of its base point.

A subgroup $g \leq h$ is a stabilizer precisely when its coset relation is effective. When all coset relations are effective, $\mathsf{Grp}(\mathcal{C})$ is exact if and only if it is regular, which is equivalent to a condition on composites of regular epimorphisms. In this way, Newman's correspondence becomes a categorical criterion for the exactness of $\mathsf{Grp}(\mathcal{C})$. We also show that regular monomorphisms of $\mathsf{Grp}(\mathcal{C})$ are stabilizers. If $\mathcal{C}$ is cartesian closed and every object embeds into an internal group, then conversely every stabilizer is regular. As a consequence, every monomorphism of cocommutative Hopf algebras is regular.

Affine group schemes also form an exact category [4, 1], but the technique does not apply as the coset relations need not be effective (the Borel subgroup of SL_2).

Keywords: Internal groups; Barr-exact categories; semi-abelian categories; cocommutative Hopf algebras; affine group schemes
Subject area: Theory of Categorical Structures
MSC2020: 18E13, 16T05, 14L15

References

[1] D. Forsman, On the semi-abelianness of affine group schemes, preprint, arXiv:2602.21060, 2026.

[2] M. Gran, F. Sterck, J. Vercruysse, A semi-abelian extension of a theorem by Takeuchi, *J. Pure Appl. Algebra* **223** (2019).

[3] K. Newman, A correspondence between bi-ideals and sub-Hopf algebras in cocommutative Hopf algebras, *J. Algebra* **36** (1975).

[4] M. Takeuchi, A correspondence between Hopf ideals and sub-Hopf algebras, *Manuscripta Math.* **7** (1972).

Friday 13 November | 11:50–12:10

The center of an associative crossed module

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Joint work with Emzar Khmaladze, Manuel Ladra and Brais Ramos Pérez.

Crossed modules of associative algebras are equivalent to internal categories in associative algebras, that is, to strict associative 2-algebras. For a crossed module $\mu\colon \mathsf{M} \to \mathsf{A}$, the multiplication of A makes the associated internal category $\mathbf{Cat}(\mathsf{M}, \mathsf{A}, \mu)$ a strict semimonoidal category. In general it has no unit, since our algebras need not be unital. We introduce and study a center in this setting, inspired by the Drinfeld center of a monoidal category and by M. Pirashvili's centers of crossed modules of groups and of Lie algebras. Its objects are triples (σ, τ, a), where $a \in \mathsf{A}$ and $\sigma\colon \mathbf{0} \Rightarrow a \cdot (-)$, $\tau\colon \mathbf{0} \Rightarrow (-) \cdot a$ are natural transformations from the zero endofunctor that are compatible with the semimonoidal structure. In other words, each object a comes with coherent null-homotopies of its left and right multiplication functors. The difference $\sigma - \tau\colon (-) \cdot a \Rightarrow a \cdot (-)$ plays the role of a half-braiding, but the pair (σ, τ) retains more information, so the construction is a Drinfeld-type center rather than the Drinfeld center itself. We prove that this center is canonically isomorphic to the internal category of an explicit crossed module $\mathbf{Z}(\mathsf{M}, \mathsf{A}, \mu)$, built from Mac Lane's bimultiplications. We also show that it carries a natural braiding satisfying the strict hexagon identities. Hence the center is a braided semimonoidal category, corresponding to a braided crossed module. It fits into a crossed square and into a homotopy fibre sequence that relates its homotopy invariants π_0 and π_1 to those of $(\mathsf{M}, \mathsf{A}, \mu)$. Associative crossed modules form a semi-abelian category, so Huq's categorical notion of center is also available. We compute the Huq center explicitly. We then show that it embeds into our center through a morphism that is an isomorphism on π_1 and injective on π_0, but is not a weak equivalence in general. Finally, we compare our center with Pirashvili's center of Lie crossed modules via the (braided) Liezation functors. We obtain a canonical comparison morphism, which preserves braidings under an injectivity condition.

Keywords: Category theory; Crossed modules; Center
Subject area: Crossed modules and category theory

References

[1] S. Mac Lane, Extensions and obstructions for rings, *Illinois J. Math.* **2** (1958), 316–345.

[2] A. Fernández-Fariña and M. Ladra, Braiding for categorical algebras and crossed modules of algebras I: Associative and Lie algebras, *J. Algebra Appl.* **19** (9) (2020), 2050176, 30 pp.

[3] M. Pirashvili, On the centre of crossed modules of Lie algebras, *J. Lie Theory* **35** (2) (2025), 359–375.

Friday 13 November | 12:10–12:30

Coalgebraic notions of simulation and bisimulation

Pouya Partow, Sergey Goncharov

School of Computer Science, University of Birmingham, United Kingdom

Presenting author: Pouya Partow

Let F be an endofunctor on a category $\mathcal{C}$. An *F-coalgebra* is a pair (X, α) consisting of an object X, the *carrier*, and a morphism $\alpha\colon X \to FX$, the *structure*. F-coalgebras model a wide range of state-based systems. For $\mathcal{C} = \mathbf{Set}$, if $F = \mathcal{P}$ is the powerset functor, then F-coalgebras are Kripke frames. If $FX = \mathcal{P}(A \times X)$ for a fixed set A, then F-coalgebras are labelled transition systems. If $FX = (\mathcal{D}X)^A$, where $\mathcal{D}$ is the distribution functor, then F-coalgebras are labeled Markov chains [3].

A bisimulation between two state-based systems is a relation that relates states with the same behaviour. A simulation is a relation that relates a state s of one system to a state t of the other only if t can do whatever s does, and possibly more. Every bisimulation is a simulation, but the converse fails in general.

Simulation and bisimulation can be defined abstractly for a pair of F-coalgebras. We refer to these definitions as *coalgebraic (bi)simulation*. Several notions of coalgebraic bisimulation have been studied, and their relationship is understood [1]. The situation for coalgebraic simulation is different. Simulation is asymmetric, and its abstract definition requires additional structure. The relationship between the different notions of coalgebraic simulation is not clarified in the literature, and neither is their relationship to the notions of coalgebraic bisimulation.

Current research on coalgebraic simulation is based on *relators* [2]. A relator for an endofunctor F on **Set** is a monotone map $\tilde{F}$ that sends every relation $r\colon X \rightarrowtail\!\!\!\!\!\!\rightarrow Y$ to a relation $\tilde{F}r\colon FX \rightarrowtail\!\!\!\!\!\!\rightarrow FY$. An endofunctor on **Set** in general has several relators. A standard example is the Barr extension $\bar{F}$ of F. A relation r is an *$\tilde{F}$-simulation* from an F-coalgebra (X, α) to an F-coalgebra (Y, β) if $r \subseteq \beta^{\mathsf{op}} \cdot \tilde{F}r \cdot \alpha$. A relator $\tilde{F}$ is *symmetric* if $\tilde{F}(r^{\mathsf{op}}) = (\tilde{F}r)^{\mathsf{op}}$ for every relation r. For a symmetric relator $\tilde{F}$, $\tilde{F}$-simulations are called *$\tilde{F}$-bisimulations*. The Barr extension $\bar{F}$ is symmetric.

Relators yield different notions of simulation and bisimulation for a single endofunctor on **Set**, but the theory of relators is restricted to **Set**. Another notion of simulation, *Aczel–Mendler simulation* [4], is defined over a general underlying category. It is a relaxation of the well-known *Aczel–Mendler bisimulation* [3]. The relationship between Aczel–Mendler simulation and relator-based simulation is not yet clear. A further goal is a notion of relator for endofunctors on an arbitrary category, giving several notions of simulation and bisimulation for one endofunctor. As a step in this direction, we propose *Hermida–Jacobs simulation*. It relaxes *Hermida–Jacobs bisimulation* [1] in the same way that Aczel–Mendler simulation relaxes Aczel–Mendler bisimulation. Its generality is limited: it generalises only the Barr extension, which is one particular relator for each functor.

In this talk, we review coalgebraic notions of simulation and bisimulation and the connections between them. We identify open questions about these connections and present the answers we have obtained so far.

Keywords: Category theory; Coalgebra; (Bi)simulations
Subject area: Coalgebra

References

[1] S. Staton, *Relating Coalgebraic Notions of Bisimulation*, Vol. 7, Logical Methods in Computer Science, 2011.

[2] S. Goncharov, D. Hofmann, P. Nora, L. Schröder, P. Wild, *Relators and notions of simulation revisited*, 40th Annual ACM/IEEE Symposium on Logic in Computer Science (LICS), 2025.

[3] J. Rutten, *The Method of Coalgebra: exercises in coinduction*, CWI, Amsterdam, 2019.

[4] J. Dubut, *Aczel–Mendler bisimulations in a regular category*, Vol. 21, Logical Methods in Computer Science, 2025.

Friday 13 November | 14:30–15:00

Descent in Probability Theory: the first steps downward

Paolo Perrone

Department of Computer Science, University of Oxford, United Kingdom

Coarse-graining, forming quotients by dropping distinctions, is a unifying idea across mathematics: identifying the endpoints of an interval yields a circle; groups are conveniently presented as quotients of free ones; sheaves and stacks emerge from gluing local data. This idea is also central to probability, where "observing" a random variable similarly quotients a sample space via the sigma-algebra it generates (and is crucial for modeling randomness as "ignorance"). Yet this quotienting procedure, in probability, has so far lacked a systematic categorical treatment.

We develop a descent theory for probability that makes this intuition precise, while respecting probabilistic practice as much as possible. On the category theory side, the theory parallels classical descent, but diverges in a few ways due to the presence of stochastic dependence (correlations). On the probability side, it unifies the three core concepts of measurability, disintegration and stochastic dominance, within a single framework, providing conceptual understanding of the relationships between random variables, statistical experiments, and inference procedures.

Partly based on joint work with Dario Stein and with Mika Bohinen.

Keywords: Category theory; probability; descent
Subject area: Category-theoretic aspects of probability
MSC2020: 60A05, 62B15 (Primary) 18C20, 18M05, 62A01 (Secondary)

References

[1] M. Di Meglio, C. Heunen, JS Lemay, P. Perrone, D. Stein, *Dagger categories of relations. The equivalence of dilatory dagger categories and epi-regular independence categories*, Submitted, arXiv:2508.01146.

[2] M. Bohinen and P. Perrone, *Categorical algebra of conditional probability*, Applied Categorical Structures 34(56), 2026

[3] D. Stein, *Random variables, independence, and categories of abstract sample spaces*, Logic in Computer Science (LICS), 2025.

[4] A. Simpson, *Category-theoretic structure for independence and conditional independence*, Electronic Notes in Theoretical Computer Science (ENTCS), 2018.

Friday 13 November | 15:00–15:30

Sheaf representation of monoidal categories, revisited

Rui Soares Barbosa

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Every small monoidal category with universal finite joins of central idempotents is monoidally equivalent to the category of global sections of a sheaf of local monoidal categories (joint work with Chris Heunen [1]). The base space is extracted from the central idempotents of the monoidal structure itself, much like the Pierce spectrum of a ring [2]. The theorem extends the sheaf representation of toposes due to Lambek and Moerdijk [3] (see also Awodey [4]) and Stone's representation of Boolean algebras [5] from the cartesian to the monoidal setting. For Hilbert modules, it relates to Takahashi's representation by continuous fields of Hilbert spaces [6], whose fibres are quotients of our stalks, much as the residue field (fibre) is obtained from a local ring (stalk).

I will present this result and sketch more recent developments, in particular a version for closed categories [7]. There, one can work with the idempotent monads $[U, -]$ instead of the central idempotents U themselves. Meets and joins of these monads always exist, given by composites and pullbacks, so one can do away with the universal joins hypothesis. Locality of the stalks is again a disjunction property, now for a Russell–Prawitz-like disjunction defined from implication [8, 9]. Implicative semilattices [10] give an example beyond the scope of the original theorem. Finally, I will briefly outline possible connections to substructural logics, where central idempotents behave like formulas admitting weakening and an invertible contraction, such as the pure assertions of intuitionistic separation logic [11, 12], and to programming languages, where monads of this kind model security levels in information-flow type systems [13].

Keywords: Monoidal categories; closed categories; sheaf representation; central idempotents
Subject area: Monoidal categories, sheaves, and categorical logic
MSC2020: 18M05, 18F20, 18D15, 18F05, 18F70

References

[1] R. S. Barbosa and C. Heunen, Sheaf representation of monoidal categories, *Advances in Mathematics* **416** (2023), 108900.

[2] R. S. Pierce, *Modules over Commutative Regular Rings*, Memoirs of the American Mathematical Society **70**, 1967.

[3] J. Lambek and I. Moerdijk, Two sheaf representations of elementary toposes, in *The L. E. J. Brouwer Centenary Symposium*, Studies in Logic and the Foundations of Mathematics **110**, North-Holland, 1982, 275–295.

[4] S. Awodey, Sheaf representations and duality in logic, in *Joachim Lambek: The Interplay of Mathematics, Logic, and Linguistics*, Outstanding Contributions to Logic **20**, Springer, 2021, 39–57.

[5] M. H. Stone, The theory of representations for Boolean algebras, *Transactions of the American Mathematical Society* **40** (1936), 37–111.

[6] A. Takahashi, Hilbert modules and their representation, *Revista Colombiana de Matemáticas* **13** (1979), 1–38.

[7] S. Eilenberg and G. M. Kelly, Closed categories, in *Proceedings of the Conference on Categorical Algebra (La Jolla, 1965)*, Springer, 1966, 421–562.

[8] B. Russell, *The Principles of Mathematics*, Cambridge University Press, 1903.

[9] D. Prawitz, *Natural Deduction: A Proof-Theoretical Study*, Almqvist & Wiksell, 1965.

[10] W. C. Nemitz, Implicative semi-lattices, *Transactions of the American Mathematical Society* **117** (1965), 128–142.

[11] J. C. Reynolds, Separation logic: a logic for shared mutable data structures, in *Proceedings of the 17th Annual IEEE Symposium on Logic in Computer Science (LICS 2002)*, IEEE, 2002, 55–74.

[12] P. W. O'Hearn and D. J. Pym, The logic of bunched implications, *Bulletin of Symbolic Logic* **5**(2) (1999), 215–244.

[13] J. Sterling and R. Harper, Sheaf semantics of termination-insensitive noninterference, in *7th International Conference on Formal Structures for Computation and Deduction (FSCD 2022)*, LIPIcs **228**, Schloss Dagstuhl – Leibniz-Zentrum für Informatik, 2022, 5:1–5:19.

Friday 13 November | 15:30–16:00

Left Kan extensions and monotone-light factorizations

João Xarez

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Let $Set^K \vdash Lan_K \colon Set^{\mathbb{B}} \to Set^{\mathbb{A}}$ be an adjunction given by left Kan extensions, and such that, for every object $A \in \mathbb{A}$, each comma category $(K \downarrow A)$ is a pseudo-filtered category, and all the morphisms in $\mathbb{B}$ are monomorphisms. The induced reflection from the adjunction above is "very-well-behaved", providing an associated monotone-light factorization system on $Set^{\mathbb{B}}$. The induced reflection $I \vdash H$ of $Set^{\mathbb{B}}$ into a full subcategory $Mono(Lan_K)$ is obtained by factorizing (componentwise) the unit morphisms of the given adjunction through surjections and injections. This new result just stated generalizes section 3 in [3]. In fact, if $\mathbb{B}$ is pseudo-filtered and $\mathbb{A} = \coprod_{i \in I} \mathbf{1}$ is the coproduct of a family of categories with just one object and one morphism (ordinal number one), it can be shown that this is a special case of the more general one above. In particular, for $\mathbb{A} = \mathbf{1}$ the left adjoint Lan_K becomes the colimit functor, and in the very simple case with $\mathbb{B} = \mathbf{2}$ (ordinal number two) the monotone-light factorization is non-trivial (that is, distinct from the reflective factorization system), as shown in section 3 in [3]; notice that, for instance, if $\mathbb{B} = \mathbf{2}$ and $\mathbb{A} = \mathbf{1} \sqcup \mathbf{1}$ then the monotone-light factorization is the same as when $\mathbb{A} = \mathbf{1}$.

Keywords: Monotone-light factorization; pseudo-filtered category; left Kan extensions; presheaves
Subject area: Category theory
MSC2020: 18A40, 18A32, 18E50

References

[1] Carboni, A., Janelidze, G., Kelly, G. M., Paré, R. *On localization and stabilization for factorization systems.* App. Cat. Struct. **5**, (1997) 1–58.

[2] Mac Lane, S. *Categories for the Working Mathematician*, 2nd ed., Springer, 1998.

[3] Xarez, J. J. *Very-well-behaved epireflections for categories of models of sketches*, Cahiers Top. Géom. Diff. Catég. LXVII(3), (2026) 355–387.

Friday 13 November | 16:30–16:50

Some categorical properties in terms of operads

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A type of algebras whose relations are multilinear can be seen as a category of algebras over an operad ([1]). From this point of view, we can understand operads as algebraic structures in which we can encode the information of the operations defined in our algebras. We are interested in understanding the link between the properties of an operad and the categorical properties of the category of algebras over it. It is known that, as soon as we have no constants different from 0 in our algebras, we are in a semi-abelian setting. We show an explicit proof of this fact.

Once that is done, we go forward and study some categorical properties in this context. In particular, we express in terms of operads the properties Normality of Higgins (NH), Smith is Huq (SH) and Algebraic Coherence (AC). We find that in the particular case of algebras over a reduced operad it is equivalent satisfying AC and both NH and SH. We also study Locally Algebraically Cartesian Closedness (LACC), finding for this context a strong condition in our operads.

Joint work with Xabier García Martínez, Sacha Ikonicoff and Tim Van der Linden. Funded by the project PID2024-155502NB-I00.

Keywords: Category theory; Operads; Algebras
Subject area: Theory of Categorical Structures
MSC2020: 18E13, 18M70

References

[1] J. Loday and B. Vallette, *Algebraic Operads*, volume 346 of *Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences]*. Springer, Heidelberg, 2012.

Friday 13 November | 16:50–17:10

Completing completions: the Karoubi envelope of the copower completion

Diogo Simm

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Free completions under limits and colimits are key constructions in category theory. In this work, motivated by computer science applications, we describe a connection between three distinct completions: the free coproduct completion (Fam-construction), the free copower completion, and the split idempotent completion (Karoubi envelope).

The main result we prove is that, for any category $\mathbb{C}$, the constant family inclusion extends to an equivalence

$$\mathrm{Kar}\big(\mathrm{Copow}(\mathbb{C})\big) \simeq \mathrm{Fam}(\mathbb{C})$$

if, and only if, $\mathbb{C}$ is Karoubi complete and every family of objects admits a *common retract host*: for a family $\{C_i\}_{i\in I}$ of objects in a category $\mathbb{C}$, morphisms

$$C_i \xrightarrow{s_i} C \xrightarrow{r_i} C_i,$$

such that $r_i \circ s_i = \mathrm{id}_{C_i}$. A sufficient condition for the existence of common retract hosts is the existence of products or coproducts.

In this talk, based on joint work with Fernando Lucatelli Nunes and Matthijs Vákár [1], we present the three relevant completions, the equivalence theorem, and sufficient conditions where it holds. Finally, we discuss the originating motivation of the work in the context of categorical semantics of programming languages.

Keywords: category theory; free completions; Karoubi envelope
Subject area: Categorical completions
MSC2020: 18A35

References

[1] F. Lucatelli Nunes, D. Simm, and M. Vákár, *Simply Typed Reverse-Mode Automatic Differentiation with Variants: Denotational Correctness via Idempotent Completion*, 2026, arXiv:2607.14453. Available at: https://arxiv.org/abs/2607.14453

Saturday 14 November | 09:30–10:00

On Graphical Freeness for Graded Linear Categories

Norihiro Yamada

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In this talk, I sketch a quotient-free graphical calculus for a free graded linear category over an arbitrary category, together with its application to a coherence problem (if time permits).

Linear categories [1] elegantly axiomatise a canonical interplay between linear and nonlinear maps recurring across logic and mathematics. Despite the intricate interaction of the comonad with the symmetric monoidal and the cartesian structures of a linear category (e.g. its standard formal theory comes with seventy knotty formal equations), I have established a *graphical calculus* for a free linear category over an arbitrary category [2]. To my knowledge, it is the first *quotient-free* presentation of the free linear category that *reduces algebra to geometry*: two morphisms are equal if and only if their graphical representatives are the same graph.

On the one hand, the graphical calculus induces quantitative information through an interpretation of paths as resource-sensitive computations. On the other hand, linear categories *graded* by semiring elements [3] refine linear categories, making the quantitative intuition behind them explicit. I then conjectured that the two quantitative structures coincide: the graphical calculus forms a free graded linear category over the base category. A positive answer to this question justifies graded linear categories as a *canonical* quantitative refinement of linear categories. I have obtained a proof sketch that solves the conjecture in the positive. In this talk, I introduce the graphical calculus, outline the proof, and, should time allow, state a necessary, sufficient graphical condition for *coherence* on graded linear categories.

Keywords: graded linear categories; graphical calculi; coherence
Subject area: graphical calculi
MSC2020: 18A50

References

[1] Maietti, M.E., Maneggia, P., De Paiva, V. and Ritter, E., 2005. *Relating Categorical Semantics for Intuitionistic Linear Logic*. Applied Categorical Structures, 13(1), pp.1-36.

[2] Yamada, Norihiro. *Graphical Freeness and Coherence for Linear Categories*. Available at https://sites.google.com/view/norihiro-yamada/publications.

[3] Gaboardi, M., Katsumata, S.Y., Orchard, D., Breuvart, F. and Uustalu, T., 2016. *Combining Effects and Coeffects via Grading*. ACM SIGPLAN Notices, 51(9), pp.476-489.

Saturday 14 November | 10:00–10:30

Exponentials for Free

Fernando Lucatelli Nunes

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Functional programming languages modulo $\beta\eta$-equations can be understood as freely generated categorical structures [1]. This classical correspondence provides a method for constructing program transformations: specify the images of primitive types and operations, and let the universal property determine the transformation. In a suitable logical-relations model, correctness likewise extends from the primitives to all programs, yielding transformations that are correct by construction.

This perspective has proved fruitful in automatic differentiation for ML-family languages [2] and in CHAD for expressive total languages [3], with developments addressing iteration and non-termination [4], simply typed implementations [5], and systematic optimisation [6]. It also points towards further extensions to probabilistic languages [7] and the compilation of value-indexed analyses beyond differentiation [8].

It is therefore natural to explore the same semantics-driven approach to defunctionalisation. We approach this problem by studying the existence of exponentials and their weak variants in freely generated categorical structures. The talk focuses on this question. I will discuss how function spaces emerge from free constructions, building on results on free extensivity and doubly-infinitary distributive categories [9, 10]. The emphasis will be on the categorical ideas and their universal properties.

The talk draws on several strands of work, including joint work with Rui Prezado and Matthijs Vákár [9], and is based primarily on ongoing joint work with Matthijs Vákár [11].

Keywords: Multicategories; free completions; cartesian closedness; defunctionalisation

References

[1] J. Lambek and P. J. Scott, *Introduction to Higher-Order Categorical Logic*, Cambridge Studies in Advanced Mathematics **7**, Cambridge University Press, 1986.

[2] F. Lucatelli Nunes and M. Vákár, Automatic differentiation for ML-family languages: correctness via logical relations, *Mathematical Structures in Computer Science* **34**(8) (2024), 747–806. doi:10.1017/S0960129524000215.

[3] F. Lucatelli Nunes and M. Vákár, CHAD for expressive total languages, *Mathematical Structures in Computer Science* **33**(4–5) (2023), 311–426. doi:10.1017/S096012952300018X.

[4] F. Lucatelli Nunes, G. Plotkin and M. Vákár, *Unraveling the iterative CHAD*, 2025, revised 2026. arXiv:2505.15002.

[5] F. Lucatelli Nunes, D. Simm and M. Vákár, *Simply Typed Reverse-Mode Automatic Differentiation with Variants: Denotational Correctness via Idempotent Completion*, DMUC Preprint 26-40, 2026. Preprint; arXiv:2607.14453.

[6] T. Smeding and M. Vákár, Efficient CHAD, *Proceedings of the ACM on Programming Languages* **8**(POPL) (2024), article 36, 1060–1088. doi:10.1145/3632878.

[7] D. Simm, F. Lucatelli Nunes and M. Vákár, *Backpropagation for Effectful Languages I: Finite Probability and Discrete Output Algebraic Effects*, DMUC Preprint 26-35, 2026. Preprint; arXiv:2607.13935.

[8] F. Lucatelli Nunes, *Compiling Value-Indexed Analyses Beyond Differentiation*. To appear.

[9] F. Lucatelli Nunes, R. Prezado and M. Vákár, Free extensivity via distributivity, *Portugaliae Mathematica* **82**(1–2) (2025), 177–204. doi:10.4171/PM/2129.

[10] F. Lucatelli Nunes and M. Vákár, Free doubly-infinitary distributive categories are cartesian closed, *Applied Categorical Structures* **34** (2026), article 53. doi:10.1007/s10485-026-09887-7.

[11] F. Lucatelli Nunes and M. Vákár, *Cartesian Closure and Defunctionalisation via Multicategories*. Manuscript in preparation.

Saturday 14 November | 10:30–11:00

Gluings of fundamental groups, via theory of descent

Rui Prezado

CMUC, University of Coimbra, Portugal

One of the main guiding principles in local-to-global arguments is the following: injective maps on local data should induce an injective map on the global data. In general, this can only be guaranteed when the local injective maps agree on intersections.

We can frame the principle as follows: given a morphism $f\colon B_1 +_A B_2 \to Y_1 +_X Y_2$ in a category $\mathcal{C}$ between two pushouts (the global data) induced by a suitable pair of monomorphisms $f_i\colon B_i \to Y_i$ for $i = 1, 2$ (the local data), what conditions guarantee that f is a monomorphism as well? As a guideline, we verify that the Van Kampen condition in an adhesive category $\mathcal{C}$ [3] will ensure the principle holds.

This problem was inspired by the study of incompressible and essential surfaces in 3-manifolds [4]. For instance, the Loop Theorem [2] provides a relationship between incompressibility and injectivity of the relevant morphisms on fundamental groups; so we focus our attention on groups homomorphisms and graphs of groups.

In our application, this argument is done via two steps: (1) a strong amalgamation property [6], to guarantee that the pushout squares are pullbacks, and (2) an argument which studies the comparison between the global objects. More specifically, the category Grp of groups satisfies the strong amalgamation property, while the comparison step is supplied by a "no-fold condition" on Bass-Serre trees [1, 5].

This abstract perspective brings forth three contributions: (1) it may be applied more generally than standard arguments, (2) it arguably better captures the geometry of the gluings involved, by specifying the required no-fold conditions, and (3) it provides a tractable set of conditions, intended for future applications.

This is joint work with F. Lucatelli Nunes and J. Nogueira.

Keywords: strong amalgamation, Van Kampen pushout, amalgamated free product, graph of groups, Bass–Serre tree, π_1-injective surface.
Subject area: Category theory, geometric topology.

References

[1] H. Bass, *Covering theory for graphs of groups*, J. Pure Appl. Algebra, 89 (1993), pp. 3–47. https://doi.org/10.1016/0022-4049(93)90085-8.

[2] J. Hempel, *3-Manifolds*, vol. 86 of Annals of Mathematics Studies, Princeton University Press, Princeton, NJ, 1976.

[3] S. Lack and P. Sobociński, *Adhesive and quasiadhesive categories*, RAIRO Theor. Inform. Appl., 39 (2005), pp. 511–545. https://doi.org/10.1051/ita:2005028.

[4] J. M. Nogueira and H. Segerman, *Incompressible surfaces in handlebodies and boundary reducible 3-manifolds*, Topology Appl., 158 (2011), pp. 551–571. https://doi.org/10.1016/j.topol.2010.11.014.

[5] J.-P. Serre, *Trees*, Springer-Verlag, Berlin, 1980.

[6] W. Tholen, *Amalgamations in categories*, Algebra Universalis, 14 (1982), pp. 391–397. https://doi.org/10.1007/BF02483940.

[7] H. Wilton, *Essential surfaces in graph pairs*, J. Amer. Math. Soc., 31 (2018), pp. 893–919. https://doi.org/10.1090/jams/901.

Saturday 14 November | 11:30–12:00

Ternary semi-direct products in algebraically coherent categories

Arnaud Duvieusart

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In the category of groups, an action of B on A can be defined equivalently as a function $B \times A \to A$ (satisfying certain axioms) or as a morphism $B \to \mathrm{Aut}(A)$. In particular, an action of a coproduct $B + C$ or a semi-direct product $B \rtimes C$ on A can easily be described by a pair of actions of B and C on A, corresponding to the induced morphisms from B and C to $[A]$.

In a *semi-abelian* category $\mathcal{C}$, an action of B on A is defined as a morphism $\xi_A^B \colon B \flat A \to A$ in $\mathcal{C}$ [1]; but a correspondence with morphisms $B \to [A]$ is only possible for *action representable* categories [2]. As a consequence, for a pair of actions (ξ_A^B, ξ_A^C) in $\mathcal{C}$, one can guarantee neither the existence nor the uniqueness of a corresponding action ξ_A^{B+C} or $\xi_A^{B \rtimes C}$.

In this talk, we will show that, in an *algebraically coherent* semi-abelian category [5], an action of $B \rtimes C$ on A is uniquely determined by a pair of actions (ξ_A^B, ξ_A^C) satisfying certain conditions. More generally, we show how *ternary semi-direct products* of A, B, C, which were previously defined by Carrasco and Cegarra in the case of groups [3] and Lie algebras [4], can be described using actions ξ_A^B, ξ_A^C, ξ_B^C and an equivariant morphism $\varphi \colon K[\xi_B^C] \to A$.

Keywords: Semi-abelian categories; actions; semi-direct products
Subject area: Categorical Algebra
MSC2020: 18E13

References

[1] F. Borceux, G. Janelidze, and G. M. Kelly. Internal object actions. *Commentationes Mathematicae Universitatis Carolinae*, 46(2):235–255, 2005.

[2] F. Borceux, G. Janelidze, and G. M. Kelly. On the representability of actions in a semi-abelian category. *Theory and Applications of Categories*, 14:244–286, 2005.

[3] P. Carrasco and A. M. Cegarra. Group-theoretic algebraic models for homotopy types. *Journal of Pure and Applied Algebra*, 75(3):195–235, 1991.

[4] P. Carrasco and A. M. Cegarra. A Dold-Kan theorem for simplicial Lie algebras. *Theory and Applications of Categories*, 32:1165–1212, 2017.

[5] A. S. Cigoli, J. R. A. Gray, and T. Van der Linden. Algebraically coherent categories. *Theory and Applications of Categories*, 30:1864–1905, 2015.

Saturday 14 November | 12:00–12:30

Cartesian closedness of lax comma categories

Maria Manuel Clementino

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We analyse cartesian closedness of the lax comma categories $Ord//X$, $Top//X$ and $Cat//X$.

This talk is based on joint work with Fernando Lucatelli Nunes, Rui Prezado e Dirk Hofmann, published at [2, 4, 1, 3].

Keywords: Lax comma categories; cartesian closed categories
Subject area: Category Theory
MSC2020: 18D15, 18A40, 18A25

References

[1] M.M. Clementino, D. Hofmann, R. Prezado, Topological lax comma categories. Order 43:3 (2026).

[2] M.M. Clementino, F. Lucatelli Nunes, Lax comma categories of ordered sets. Quaestiones Mathematicae 46 (S1) (2023) 145-159.

[3] M.M. Clementino, F. Lucatelli Nunes, Cartesian closedness of lax comma categories. DMUC Preprint 26-59. arXiv 2611.01184

[4] M.M. Clementino, F. Lucatelli Nunes, R. Prezado, Lax comma categories: cartesian closedness, extensivity, topologicity, and descent. Theory Appl. Categ. 41 (16) (2024), 516-530.

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