

Monads on the category of metric spaces

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Abstract

The category $\mathbf{Met}$ of metric spaces and nonexpanding maps is locally countably presentable in the enriched sense. It follows from the work of Kelly and Power that for countably accessible monads T on $\mathbf{Met}$ the category of algebras can be described as an equational category of S -algebras for a generalized signature S with countable arities. This has been recently sharpened by Rosický [R] who used quantitative equations between terms s, t . These equations are indexed by a rational number e , and an algebra satisfies this if and only if every computation of s and t results in elements of distance at most e .

Mardare et al. work with classical finitary signatures S and with omega-basic equations [M]. This means quantitative equations with specified conditions on the distances of pairs of variables. These authors have exhibited in a series of articles several important examples of omega-varieties, i.e. classes of S -algebras presented by omega-equations. Can omega-varieties be characterized as monadic categories over $\mathbf{Met}$? We do not have an answer. However, by moving from $\mathbf{Met}$ to the category of $\mathbf{UMet}$ ultrametric spaces, we present such a characterization of omega-varieties: they are precisely the monadic categories for enriched monads T on $\mathbf{Umet}$ preserving surjective morphisms and directed colimits of split subobjects.

References: [M] R. Mardare, P. Panangaden, and G. Plotkin. Quantitative algebraic reasoning. In Proceedings of LICS 2016, pages 700-709 [R] J. Rosický, Metric monads, arXiv:2012.14641v5