

The theory of exactness properties: recent developments and work in progress

Zurab Janelidze

Stellenbosch University, South Africa

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Abstract

By the “theory of exactness properties” we mean a subject that is in similar relation to the theories of semi-abelian categories, abelian categories, etc., as universal algebra is to group theory, module theory, etc. These classes of categories (semi-abelian, abelian, etc.) are identified by properties that express how limits and colimits behave, either on their own or in relation to each other. Since the class of abelian categories was historically the first example to be considered, where in many cases the properties of limits and colimits can be reduced to properties of exact sequences, and since their study has inspired the study of many other similar classes of categories, such properties in general are called “exactness properties”. The term “exactness property” may refer to a property closely linked with some property of an abelian category, or its generalization, as well as to a property that is as far from the concept of an abelian category as possible, such as the property of existence of a subobject classifier, for example, which is one of the main properties defining a topos. Even if abelian categories and toposes appear to be very different from each other, one should keep in mind that: first of all, they are both Barr exact categories (an exactness property), and secondly, the dual of a topos has many features of an abelian category - for instance, it is Bourn protomodular (an exactness property). Modern category theory knows many exactness properties. Although many of these arise as generalizations of properties defining abelian categories or toposes, they are of independent interest, with many examples not covered by abelian categories or toposes, and also many theorems that are not applicable to these two types of categories. This brings one to two questions: (1) First of all, what is an exactness property, formally? (2) Secondly, are there any general theorems that apply simultaneously to different classes of categories defined by exactness properties? Investigation of these two questions is the subject of the “theory of exactness properties”. In this talk we present some recent developments and some work in progress that address these questions.