
A general approach to pointfree T_0 spaces

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Date & Time: August 19, 2025 (Tuesday) — 12:00 PM (Coimbra time)

Place: Google Meet

Abstract. The classical dual adjunction between frames and spaces, given by the functors $O: \mathbf{Top} \rightarrow \mathbf{Frm}$ and $\text{pt}: \mathbf{Frm} \rightarrow \mathbf{Top}$, restricts to a dual equivalence between sober spaces and spatial frames. Another pointfree approach to the study of spaces is the so-called TD duality, a similar dual adjunction where the fixpoints are the TD spaces. The two dualities are incompatible in that the two spectrum functors are different. Recently, a pointfree approach to spaces whose fixpoints are all T_0 spaces was introduced. The approach is inspired by Raney duality. In this setting, we study the category of Raney extensions and show that we have a dual adjunction extending (O, pt) and capturing all T_0 spaces. By suitably restricting on objects we recover both the classical and the TD duality. Other pointfree notions of T_0 spaces are given by strictly zero-dimensional biframes and McKinsey–Tarski algebras. Aiming to understand the relationship between these three different settings, we introduce and study an abstract notion of pointfree T_0 space. We study the situation where we have a suitable forgetful functor $U: \mathcal{C} \rightarrow \mathbf{Frm}$ and a dual adjunction $O_{\mathcal{C}}: \mathbf{Top} \rightarrow \mathcal{C}$ and $\text{pt}_{\mathcal{C}}: \mathcal{C} \rightarrow \mathbf{Top}$, whose fixpoints are the T_0 spaces and which is, in a sense that will be made precise, compatible with the adjunction (O, pt) between frames and spaces. By studying the fibers of U , we are able to study various aspects of T_0 spaces abstractly. In all pointfree notions of T_0 spaces mentioned above, the pointfree sober spaces are the top elements of the fibers, and the TD ones are the bottom elements.