N-fold groupoids and n-groupoids in regular Mal'tsev categories

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Abstract. The category 2-Grpd($\mathscr{C}$) of internal 2-groupoids in a regular Mal'tsev category $\mathscr{C}$ with finite colimits is a Birkhoff subcategory of the category Grpd^2($\mathscr{C}$) of internal double groupoids in $\mathscr{C}$, and the left adjoint of the inclusion functor of 2-Grpd($\mathscr{C}$) into Grpd^2($\mathscr{C}$) allows for an easy explicit descripton [4]. Using this one can show that also the category n-Grpd($\mathscr{C}$) of internal n-groupoids in $\mathscr{C}$ [1] is a Birkhoff subcategory of the category Grpd^n($\mathscr{C}$) of internal n-fold groupoids in $\mathscr{C}$. Since n-Grpd($\mathscr{C}$) is also coreflective in Grpd^n($\mathscr{C}$) (as soon as $\mathscr{C}$ is finitely complete), the result in [4] that Grpd^n($\mathscr{C}$) is action representable [2] implies that also n-Grpd($\mathscr{C}$) is action representable whenever $\mathscr{C}$ is a semi-abelian algebraically coherent category with normalizers that is action representable.

[1] D. Bourn, *La tour de fibrations exactes des n-catégories*, Cah. Topol. Géom. Différ. Catég. 25 :4 (1984), 327-351.

[2] F. Borceux, G. Janelidze and G.M. Kelly, *Internal object actions*, Comm. Math. Univ. Carolin. 46:2 (2005), 235-255.

[3] N. Egner and M. Gran, *Double groupoids and 2-groupoids in regular Mal'tsev categories*, App. Categ. Structures 33:27 (2025).

[4] M. Gran and J.R.A. Gray, *Action representability of the category of internal groupoids*, Theory Appl. Categ. 37:1 (2021), 1-13.