

## **Esakia duality and its connection with pointfree Pervin spaces**

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### **Abstract**

Frames are complete Heyting algebras, but frame maps need not preserve the Heyting arrow. Those maps that preserve it are called open maps. Using the classical adjunction between frames and spaces, one can show that frames with open maps are dual to topological spaces with maps satisfying a property strictly weaker than topological openness, called the interior property.

This result can be refined. We can define a category  $\mathbf{PHFrm}$  whose objects are Pervin-Heyting frames, i.e. pairs  $(L, H)$  where  $L$  is a frame and  $H$  a Heyting subalgebra of  $L$  such that it generates  $L$ , and morphisms are frame maps which are only required to preserve the Heyting arrow in the designated subalgebra, and which preserve the property of elements being in the designated subalgebra. As duals of Pervin-Heyting frames we have the objects of the category  $\mathbf{PHTop}$ , that is Pervin-Heyting spaces. These are pairs  $(X, S)$ , where  $X$  is a set and  $S$  is a sublattice of its powerset such that it is a Heyting algebra. The morphisms in  $\mathbf{PHTop}$  are maps which satisfy a generalization of the interior property.

The category of Heyting algebra is equivalent to the full subcategory of  $\mathbf{PHFrm}$  determined by the objects which are in a certain sense complete. Similarly, the category of Esakia spaces is equivalent to the subcategory of  $\mathbf{PHTop}$  consisting of the complete Pervin-Heyting spaces. Under these equivalences Esakia duality is given by a restriction of the Pervin-Heyting duality along full inclusions.