

Topology for internal pre-orders: focus on complete regularity

Walter Tholen

York University, Canada

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Abstract

Treating the ultrafilter monad for sets as a role model, under modest hypotheses on the two parameters involved, we consider some topological notions for T-pre-orders, internal to a category $\mathcal{C}$ that comes equipped with a monad T . We call them T-spaces in $\mathcal{C}$, present a rudimentary theory for them, and then focus on complete regularity. The full subcategory of T-spaces with this property is, like the category of all T-spaces, fibred over $\mathcal{C}$, but distinguishes itself by a universal property in $\mathbf{CAT}/\mathcal{C}$ vis-à-vis the monadic subcategory of compact Hausdorff T-spaces. The generality of the setting offers a broad array of example situations.

The talk is based in part on recent work with Amir Homayoun Nejah. Some references follow, but in the talk we will not assume any prior knowledge of the subject.

[01] M. Barr: Relational algebras, *SLNM* 137, 1970. [02] R. Börger: Coproducts and ultrafilters, *JPAA* 46, 1987. [03] A. Burroni: T-catégories (catégories dans un triple), *Cahiers* 12, 1971. [04] M.M. Clementino and W. Tholen: Metric, topology and multicategory-a common approach, *JPAA* 179, 2003. [05] D. Hofmann, G.J. Seal and W. Tholen (eds.): *Monoidal Topology*, Cambridge University Press, 2014. [06] G. Janelidze and W. Tholen: Extended Galois theory and dissonant morphisms, *JPAA* 143, 1999. [07] T. Leinster: Codensity and the ultrafilter monad, *TAC* 28, 2013. [08] E.G. Manes: Compact Hausdorff objects, *Top. Appl.* 4, 1974. [09] A.H. Nejah and W. Tholen: A categorical review of complete regularity, in preparation. [10] R.J. Perry: Completely regular relational algebras, *Cahiers* 17, 1976. [11] W. Tholen and L. Yeganeh: The comprehensive factorization of Burroni's T-functors, *TAC* 36, 2021.