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Norms for vectors, linear operators and morphisms of categories, and Cauchy convergence for them

Abstract

In this talk we explore Lawvere's notion of normed category through the lens of easily understood small and large example categories. Cauchy convergence of sequences in such categories gets presented as a natural extension of familiar concepts taught in Calculus and Functional Analysis, but has broader impact, also because we allow norms to have values in an arbitrary quantale, rather than just in the real numbers. Our concept fits with the notion of weighted colimit of enriched category theory. There is therefore a natural notion of Cauchy cocompletion of a normed category, the existence proof for which draws on general results from that theory. As an application, Banach's Fixed Point Theorem gets extended almost verbatim from metric spaces to normed categories, and it then adds to the large array of results in functorial fixed point theory as studied predominantly by computer scientists.

(Based on joint work with M.M. Clementino and D. Hofmann)